Periodic Motion 周期运动

Oscillation of a spring 弹簧震动

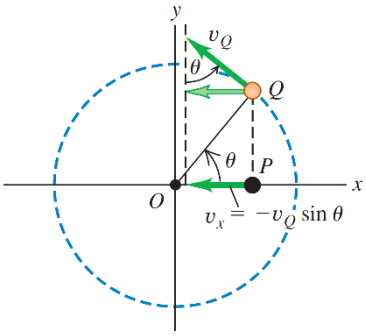
Restoring force 恢复力 $F_x = -kx$

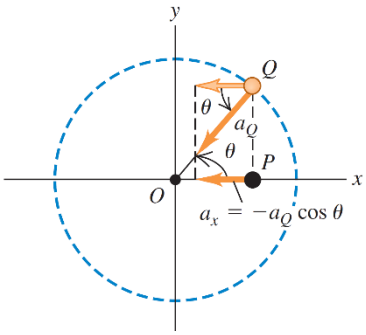
加速度 $a_x = -\frac{k}{m}x$

周期和频率 $T = \frac{1}{f}$

角频率 $\omega = 2\pi f = \frac{2\pi}{T}$

Simple Harmonic Motion 简谐震动





Let $\omega = \sqrt{\frac{k}{m}}$ $\omega = \sqrt{\frac{k}{m}}$是系统固有属性

运动学方程 $x(t) = Acos(\omega t + \emptyset)$

速度 $v = \frac{dx}{dt} = -\omega Asin(\omega t + \emptyset)$

$v_{max} = \omega A = \sqrt{\frac{k}{m}}\mathrm{A}$

加速度 $a = \frac{dv}{dt} = -\omega^2 Acos(\omega t + \emptyset) = -\omega^2 x$

$a_{max} = \omega^2 A = \frac{k}{m}\mathrm{A}$

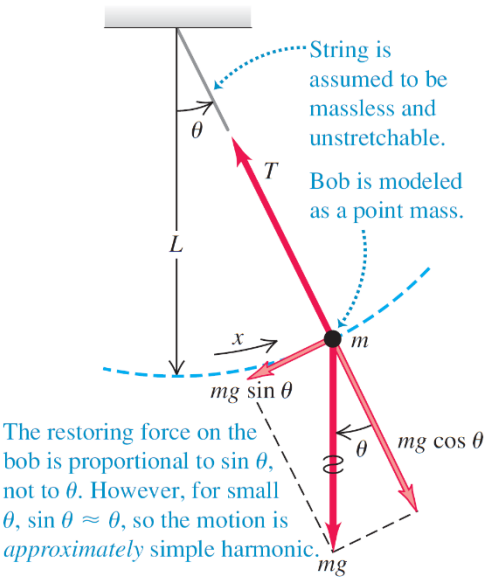
势能 $\mathrm{U} = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \emptyset)$

动能 $\mathrm{K} = \frac{1}{2}mv^2 = \frac{1}{2}kA^2 sin^2(\omega t + \emptyset)$

势能+动能=常数 $\mathrm{U} + \mathrm{K} = \frac{1}{2}kA^2$

The Simple Pendulum 单摆运动

An idealized simple pendulum



Restoring force: $F_\theta = -mg\sin\theta \approx -mg\theta$

$F_\theta = -mg\theta = -mg\frac{x}{L} \Rightarrow \mathrm{k} = -\frac{\mathrm{mg}}{L}$

Angular Frequency: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{m}}$

Frequency: $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

Period: $T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$

Exercise 17: The velocity of a simple harmonic oscillator is given by the equation $\mathrm{v} = 3\cos(2t + \pi/4)$, where v is in m/s and t is in second. What is its position at $t = 5\pi/8$ if its position is $\mathrm{x} = 3/2$ (m) at $\mathrm{t} = \pi/8$ (s)?

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$$x = \int v\,dt + c = \int 3\cos(2t + \pi/4)\,dt + c = \frac{3}{2}\sin(2t + \pi/4) + c$$

$$\frac{3}{2} = \frac{3}{2}\sin(\frac{2\pi}{8} + \frac{\pi}{4}) + c = \frac{3}{2}\sin(\frac{\pi}{2}) + c = \frac{3}{2} + c \quad \Rightarrow c = 0$$

$$x = \frac{3}{2}\sin(2t + \pi/4)$$

$$x\left(\frac{5\pi}{8}\right) = \frac{3}{2}\sin\left(2 \times \frac{5\pi}{8} + \frac{\pi}{4}\right) = \frac{3}{2}\sin\frac{3\pi}{2} = \frac{-3}{2} \quad (m)$$